ΛΥΣΗ

α) Έχουμε:

$\alpha^2 = \left(1+2\sqrt{2}\right)^2 = 1^2 + 2\cdot 2\sqrt{2}\cdot 1 + \left(2\sqrt{2}\right)^2 = 1 + 4\sqrt{2} + 2^2\left(\sqrt{2}\right)^2 = 1 + 4\sqrt{2} + 8 = 9 + 4\sqrt{2}$ και

$\beta^2 = \left(\sqrt{2}-2\right)^2 = \left(\sqrt{2}\right)^2 - 2\cdot\sqrt{2}\cdot 2 + 2^2 = 2 - 4\sqrt{2} + 4 = 6 - 4\sqrt{2}$.

Οπότε $\alpha^2 + \beta^2 = 9 + 4\sqrt{2} + 6 - 4\sqrt{2} = 15$.

β) Είναι $\alpha = 1 + 2\sqrt{2} > 0$ και $\beta = \sqrt{2} - 2 < 0$, οπότε:

$\sqrt{\alpha^2} + 2\sqrt{\beta^2} = |\alpha| + 2|\beta| = \alpha + 2(-\beta) = 1 + 2\sqrt{2} + 2\left(2 - \sqrt{2}\right) = 1 + 2\sqrt{2} + 4 - 2\sqrt{2} = 5$.